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ON SOME PROPERTIES OF FOUR CIRCLES INSCRIBED IN
ONE AND CIRCUMSCRIBED ABOUT ANOTHER.

BY CHRISTINE LADD, UNION SPRINGS, N. Y.

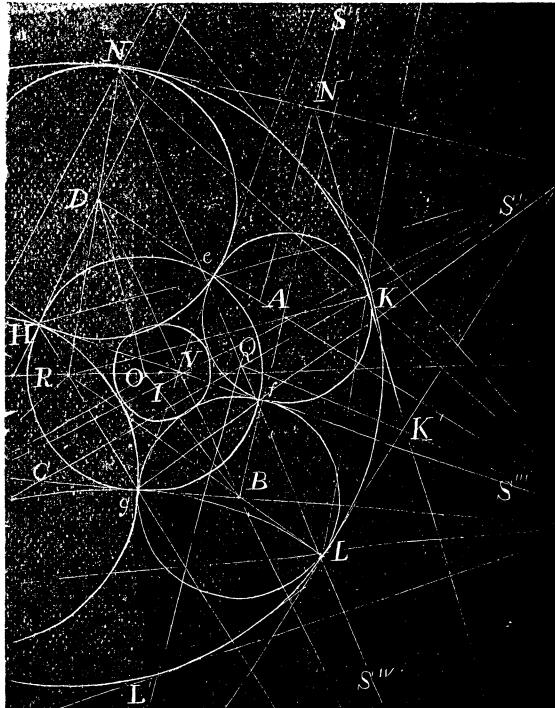
LET the circles A, B, C, D be tangent to the circle R internally and to the circle I externally.

I. The circles A, B, C, D have a common axis of similitude; for, R and I being a pair of tangent circles, their radical axis is coincident with the axis of similitude of any three, and therefore all of the circles A, B, C, D . (Salmon's *Conic Sections*, page 109, note.)

If E, F, G, H are the points of contact of these circles with each other, then HE and GF intersect in S' , the external centre of similitude of the circles A and C , and HG & EF intersect in S'' , the external centre of similitude of D and B .

II. The circles A, B, C, D have a common radical centre; for the perpendiculars to the sides of the quadrilateral $ABCD$ at the points E, F, G, H intersect, each two, on the line RI (Salmon, loc. cit.) and therefore all four in the same point, O . The common orthogonal circle, O , is then inscribed in $ABCD$, touching it in the points E, F, G, H .

III. Since tangents to A and D , at the points H and F respectively, meet on the radical axis of A and D (namely, in the point O), it follows that H and F are antihomologous points on the circles D and A , and hence the line HF passes through S'' . In the same way it may be shown that GE passes through S' . This is the essential quality of the quadrilaterals $ABCD$ and $EFGH$.



IV. The diagonals of $ABCD$ and $EFGH$ meet in a point, V , the polar of $S'S''$ and therefore on the line RI (Booth, *New Geometrical Methods*, Vol. II., Art. 245). This includes Problem 187 of the ANALYST.

V. If $KLMN$ is the quadrilateral formed by joining the points of contact on the circle R , its sides and diagonals pass through the points already determined on the line $S'S'''$; for N and K being centres of similitude of the circles A , D and R , NK passes through the external centre of similitude of A and D . The same is true for the corresponding circumscribed quadrilateral, $K'L'M'N'$, and the diagonals of these two quadrilaterals meet in a point, Q , which with the points $S'S^{iv}$ forms a triangle self-conjugate with respect to the circle R . In the same way it may be shown that $S'VS^{iv}$ is self-conjugate with respect to the circle I .

VI. The points S'', S', S''', S^{iv} form an harmonic range (Catalan, *Theore'mes et Proble'mes*, III., 49).

VII. Since $BG = BL$ and $CG = CM$, L' is the centre of an exscribed circle to the triangle BRG and that circle is tangent to CB at G .

VIII. The circles about L' , M' , N' , K' as centres, and tangent to the sides of $ABCD$, form another set of four circles circumscribed about one, the circle O , and inscriptible in another, since the quadrilaterals $L'M'N'K'$ and $LMNK$ have the diagonals of each passing through the intersections of sides of the other, and are corresponding circumscribed and inscribed quadrilaterals to the circle R (Proposition III).

The points A , B , C , D are evidently on an ellipse to which the points R and I are foci. They are concyclic only when O and V (and therefore R and I) coincide, for only then is VS'' , or HF , perpendicular to EG .

IX. To inscribe in a given circle four circles tangent to each other and to another circle:—

Take any triangle, as $S'QS^{iv}$, self-conjugate with respect to the given circle, R . At K , L , M , N , the points where its sides cut the circle, R , draw tangents to R , and with their intersections as centres describe arcs orthogonal to R . From the points where the diagonals of $KLMN$ meet $S'S^{iv}$ draw tangents to these arcs; their intersections will be the centres of the required circles. The proof of this construction readily follows from the properties already obtained.

CUBIC EQUATIONS.

BY HENRY HEATON, SABULA, IOWA.

If a and b be supposed to be either positive or negative, it is well known that every cubic equation may be reduced to the form

$$x^3 - 3ax = 2b. \quad (1)$$